Localization-delocalization transition of disordered d-wave superconductors in class CI

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A lattice model for disordered d-wave superconductors in class CI is reconsidered. Near the band-center, the lattice model can be described by Dirac fermions with several species, each of which yields WZW term for an effective action of the Goldstone mode. The WZW terms cancel out each other because of the four-fold symmetry of the model, which suggests that the quasiparticle states are localized. If the lattice model has, however, symmetry breaking terms which generate mass for any species of the Dirac fermions, remaining WZW term which avoids the cancellation can derive the system to a delocalized strong-coupling fixed point.

Dirty superconductors have attracted much interest, since they provide wider universality classes of disordered systems [1–3]. In particular, it is quite interesting to ask what the universality class of the disordered d-wave superconductors is [4–7]. A remarkable property of this unconventional superconductors is that near the band center, quasiparticle states can be described by Dirac fermions [8]. Such a description enables us, for example, to relate d-wave superconductors with the quantum Hall effect and to predict a new spin phase called spin quantum Hall fluid [9]. They should also produce the well-known effect of chiral anomaly [10,11] to d-wave superconductors. Recently, Senthil *et al* [6] have studied disordered d-wave superconductors with spin rotational symmetry and reached the conclusion that all states are localized. One knows, however, that the WZW term due to chiral anomaly plays a crucial role in two dimensional critical phenomena [11]. Therefore, it is quite important to take the WZW term missing in [6] into account, or to answer the question why it vanishes if it does not exist.

In this paper, we reconsider disorder effects on the quasiparticle properties of the d-wave superconductors in the class CI (those with spin rotational and time-reversal symmetries) using a replica technique. It is shown that each Dirac fermion associated with four nodes creates the WZW term. It turns out that they cancel each other and the resultant nonlinear sigma model suggests localization of the quasiparticles in dirty d-wave superconductors, as was shown by Senthil *et al* [6]. It should be stressed, however, that the WZW term is potentially realizable and the cancellation is accidental: It is due to the four-fold symmetry of the model. Therefore, if such symmetry is broken, the system flows to the strong-coupling fixed point described by the WZW model.

Let us begin with a lattice Hamiltonian for singlet superconductors [4,6,7],

$$H=\sum_{i,j}\left(t_{ij}\sum_{\sigma}c^\dagger_{i\sigma}c_{j\sigma}+\Delta_{ij}c^\dagger_{i\uparrow}c^\dagger_{j\downarrow}+\Delta^*_{ij}c_{j\downarrow}c_{i\uparrow}\right). \quad (1)$$

We can choose real and symmetric matrices $t_{ij}=t_{ji}$ and $\Delta_{ij}=\Delta_{ji}$ taking account of the hermiticity as well as the spin-rotational and the time-reversal symmetries. In the absence of randomness, we choose the following parameters for a pure Hamiltonian H_0 with d-wave symmetry

$$t_{j,j\pm\hat{x}}=t_{j,j\pm\hat{y}}=-t_0,\quad \Delta_{j,j\pm\hat{x}}=-\Delta_{j,j\pm\hat{y}}=\Delta_0, \quad (2)$$

where $\hat{x}=(1,0)$ and $\hat{y}=(0,1)$. Moreover, consider introducing small terms H_1 to break the CI symmetry,

$$\Delta_{j,j+\hat{x}\pm\hat{y}}=\Delta_{j+\hat{x}\pm\hat{y},j}=\pm\frac{i\Delta_1}{4},\quad \Delta_{j,j}=i\Delta_1. \quad (3)$$

It will be shown momentarily that this parameter controls the localization-delocalization transition of the present model.

The pure Hamiltonian H_0 has four nodes, where gapless quasi-particle excitations exist [4]. Therefore, we can firstly take the continuum limit around the nodes of H_0 and next incorporate the continuum expression of H_1, provided that H_1 is small. The lattice operators are then described by the continuum slowly-varying fields near the band center as [6],

$$\begin{aligned}c_{j\uparrow}/a&\sim i^{j_x+j_y}\psi^1_{\uparrow1}(x)-i^{-j_x-j_y}\psi^1_{\downarrow2}(x)\\&\quad+i^{-j_x+j_y}\psi^2_{\uparrow1}(x)-i^{j_x-j_y}\psi^2_{\downarrow2}(x),\\c_{j\downarrow}/a&\sim i^{j_x+j_y}\psi^{1\dagger}_{\downarrow1}(x)+i^{-j_x-j_y}\psi^{1\dagger}_{\uparrow2}(x)\\&\quad+i^{-j_x+j_y}\psi^{2\dagger}_{\downarrow1}(x)+i^{j_x-j_y}\psi^{2\dagger}_{\uparrow2}(x),\end{aligned} \quad (4)$$

where a is a lattice constant, $x=aj$, and two kinds of lower indices of the field ψ are referred to as spin, left-right (LR) movers, respectively, and upper index as node. Namely, the field variable $\psi(x)$ lives in the space $V=\boldsymbol{C}^2\otimes\boldsymbol{C}^2\otimes\boldsymbol{C}^2$. The pure Hamiltonian in the continuum limit [4,6] is then $H_0=\int d^2x\psi^\dagger(\mathcal{H}_0+\mathcal{H}_1)\psi$ with

$$\mathcal{H}_0=\begin{pmatrix}-\gamma_\mu i\partial_\mu & \\ & (x\leftrightarrow y)\end{pmatrix}, \quad (5)$$

where the coordinates have been transformed as $x,y\to\frac{\pm x+y}{\sqrt{2}}$. The explicit matrix in Eq. (5) denotes the node space, and matrices γ_μ belong to the other space of V, calculated initially as $\gamma_1=v_F1_2\otimes\sigma_3$ and $\gamma_2=v_\Delta1_2\otimes\sigma_1$, where $v_F=2\sqrt{2}t_0a$ and $v_\Delta=2\sqrt{2}\Delta_0a$. It may be more convenient to choose

$$\gamma_1 = v_F 1_2 \otimes \sigma_2, \quad \gamma_2 = -v_\Delta 1_2 \otimes \sigma_1, \tag{6}$$

via suitable rotation in LR-space of V. In this basis, $\mathcal{H}_1$ is given by

$$\mathcal{H}_1 = \begin{pmatrix} 0 & \\ & m 1_2 \otimes \sigma_3 \end{pmatrix}, \tag{7}$$

where $m = \Delta_1$. Namely, H_1 yields asymmetric mass term in the continuum Hamiltonian. For the time being we neglect it, but it will be shown that vanishing m leads to localization whereas finite m drives the system to delocalization.

The spin-rotational and time-reversal symmetries of the lattice model translate, respectively, into the continuum model as

$$\begin{array}{ll} \mathcal{H} = -\mathcal{C}\mathcal{H}^{\mathrm{t}}\mathcal{C}^{-1}, & \mathcal{C} = i\sigma_2 \otimes \sigma_1 \otimes 1_2, \\ \mathcal{H} = -\mathcal{T}\mathcal{H}\mathcal{T}^{-1}, & \mathcal{T} = 1_2 \otimes \sigma_3 \otimes 1_2, \end{array} \tag{8}$$

where t means the transpose. The total Hamiltonian density is given by $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\mathrm{d}}$, where $\mathcal{H}_{\mathrm{d}}$ is disorder potential satisfying Eq. (8). It is stressed that we take account of all kinds of disorder potentials satisfying Eq. (8). This implies that the summation with respect to i, j in Eq. (1) should be over on-site, nearest-neighbor, and diagonal-second-neighbor pairs. Namely, we can achieve "maximum information entropy" for the Dirac Hamiltonian with the symmetries (8) when we introduce not only on-site disorder potentials but also disordered hopping and off-diagonal pairing for the lattice model in Eq. (1). Actually Ludwig *et al* have derived generic Dirac Hamiltonian for the integer quantum Hall transition considering lattice model in a similar situation [8]. It may be straightforward to explicitly calculate disorder potentials but tedious to average over them, because there are no less than twenty independent potentials satisfying Eq. (8). The key point for the ensemble average is that if the model has "maximum entropy", we can use the technique developed by Zirnbauer [2].

To study one-quasiparticle properties of the model, we introduce the Green function $G_{aa'}(x, x'; i\epsilon) = \langle x, a|(i\epsilon - \mathcal{H})^{-1}|x', a'\rangle$, where index a denotes the set of spin, LR, and node species in the space V and $|x, a\rangle = \psi_a^\dagger(x)|0\rangle$. Especially, we need

$$\begin{aligned} G(x) &= \sum_a G_{aa}(x, x; i\epsilon), \\ K(x, x') &= \sum_{a,a'} G_{aa'}(x, x'; i\epsilon) G_{a'a}(x', x; -i\epsilon) \end{aligned} \tag{9}$$

to compute the DOS and the conductance of the quasiparticle transport [12]. Some notations are convenient for this purpose. The introduction of replica for the field ψ enables us to express the generating functional of these Green functions by path integrals: $\psi_a \to \psi_{a\alpha}$ and $\psi_a^* \to \psi_{\alpha a}^*$, where a and α are indices denoting V and the replica space $W_{\mathrm{r}} = \boldsymbol{C}^n$, respectively. The fields ψ and ψ^* have been converted into matrix fields, which makes it simpler to define an order parameter field. It should be stressed that the fields ψ and ψ^* are completely independent variables. Lagrangian density is then described symbolically as $\mathcal{L} = -\mathrm{tr}_{W_{\mathrm{r}}} \psi^\dagger (i\epsilon - \mathcal{H}) \psi$, where $\mathrm{tr}_{W_{\mathrm{r}}}$ is the trace in the replica space and the summation over the indices a of the V space is implied according to the rule of the matrix product. It should be noted again that $\psi^\dagger$ is independent of ψ. Moreover, we introduce an auxiliary space [2] to reflect the symmetries in the V space (8) to an auxiliary field introduced later [See Eq. (15)], $W_{\mathrm{r}} \to W = W_{\mathrm{r}} \otimes W_{\mathrm{a}}$ with $W_{\mathrm{a}} = \boldsymbol{C}^2 \otimes \boldsymbol{C}^2$, which are associated with the spin-rotational and the time-reversal symmetries, respectively. Fermi fields are now denoted by $\widetilde{\Psi}_{\alpha i}$ and $\Psi_{i\alpha}$. One of simpler choices is

$$\begin{aligned} &\Psi = (\psi_+, \psi_-), \quad \psi_\pm = \mathcal{T}_\pm \widetilde{\psi}, \\ &\widetilde{\Psi} = \begin{pmatrix} \bar{\psi}_+ \\ \bar{\psi}_- \end{pmatrix}, \quad \bar{\psi}_\pm = \widetilde{\psi}^\dagger \mathcal{T}_\mp, \end{aligned} \tag{10}$$

where $\mathcal{T}_\pm = (1 \pm \mathcal{T})/2$ serves as a projection operator ($\mathcal{T}_+ + \mathcal{T}_- = 1_2 \otimes 1_2 \otimes 1_2$, $\mathcal{T}_\pm^2 = \mathcal{T}_\pm$, and $\mathcal{T}_+\mathcal{T}_- = 0$) into each chiral component of $\mathrm{Sp}(n) \times \mathrm{Sp}(n)$ symmetry, as we shall see later, and $\widetilde{\psi} = \frac{1}{2}(\psi_1, -i\psi_2)$ with $\psi_{1,2} = \psi \pm i\mathcal{C}^{-1}\psi^*$. The newly introduced fields Ψ and $\widetilde{\Psi}$ are subject to [2],

$$\begin{array}{ll} \widetilde{\Psi} = \gamma \Psi^{\mathrm{t}} \mathcal{C}^{-1}, & \Psi = \mathcal{C} \widetilde{\Psi}^{\mathrm{t}} \gamma^{-1}, \\ \widetilde{\Psi} = -\pi \widetilde{\Psi} \mathcal{T}^{-1}, & \Psi = \mathcal{T} \Psi \pi^{-1}. \end{array} \tag{11}$$

Matrices γ and π are defined in the W space by

$$\begin{aligned} \gamma &= 1_n \otimes i\sigma_2 \otimes 1_2 \equiv \gamma_0 \otimes 1_2, \\ \pi &= 1_n \otimes 1_2 \otimes \sigma_3. \end{aligned} \tag{12}$$

The identity $\mathrm{tr}_W (i\epsilon\omega\widetilde{\Psi}\Psi - \widetilde{\Psi}\mathcal{H}\Psi) = \mathrm{tr}_{W_{\mathrm{r}}} \psi^\dagger (i\epsilon - \mathcal{H})\psi$, where $\omega = 1_n \otimes \sigma_2 \otimes \sigma_1$, leads to the generating functional $\mathcal{Z} = \int \mathcal{D}\Psi \mathcal{D}\widetilde{\Psi} e^{-S}$ with

$$S = -\int d^2x \,\mathrm{tr}_W \left(i\epsilon\omega\widetilde{\Psi}\Psi - \widetilde{\Psi}\mathcal{H}\Psi + J\widetilde{\Psi}\Psi \right). \tag{13}$$

Assume that disorder potential $\mathcal{H}_{\mathrm{d}}$ obeys the Gaussian distribution $P[\mathcal{H}_{\mathrm{d}}] = \exp\left(-\frac{1}{2g}\mathrm{tr}_V \mathcal{H}_{\mathrm{d}}^2\right)$. Then ensemble average over disorder is quite simple. The procedure is as follows [2]: The disorder potentials are integrated out by using the identity $-\frac{1}{2g}\mathrm{tr}_V \mathcal{H}_{\mathrm{d}}^2 + \mathrm{tr}_V \mathcal{H}_{\mathrm{d}} \Psi\widetilde{\Psi} = -\frac{1}{2g}(\mathrm{tr}_V \mathcal{H}_{\mathrm{d}} - g\Psi\widetilde{\Psi})^2 + \frac{g}{2}\mathrm{tr}_V (\Psi\widetilde{\Psi})^2$. It turns out that the integration over $\mathcal{H}_{\mathrm{d}}$ is automatic because $\Psi\widetilde{\Psi}$ satisfy the same symmetries as those of $\mathcal{H}_{\mathrm{d}}$ due to Eq. (11). This is actually a merit to consider the disorder potentials with maximum entropy. If disordered hopping and off-diagonal pairing of the lattice model are neglected and on-site disorder potentials are merely taken into account, some other conditions should be imposed on $\mathcal{H}_{\mathrm{d}}$ and therefore on the fields Ψ and $\widetilde{\Psi}$. Now we

have interaction terms of fermions due to ensemble average. Note the identity $\mathrm{tr}_V(\Psi\widetilde{\Psi})^2 = -\mathrm{tr}_W(\widetilde{\Psi}\Psi)^2$. Then, the four fermi interactions are decoupled via auxiliary matrix (order parameter) field defined in the W space. To be concrete, add the following term into the action, $\frac{1}{2g}\mathrm{tr}_W(Q + g\widetilde{\Psi}\Psi - \omega)^2$, which is actually a constant after integration over Q. Then we reach an effective Lagrangian density,

$$\mathcal{L} = -\mathrm{tr}_W\left[\frac{1}{2g}\left(Q^2 - 2i\epsilon\omega Q\right) + Q\widetilde{\Psi}\Psi - \widetilde{\Psi}\mathcal{H}_0\Psi\right]. \quad (14)$$

Here we have set $J = 0$ for simplicity. Notice that the anti-Hermitian auxiliary field $Q = -Q^\dagger$ is subject to

$$Q = -\gamma Q^{\mathrm{t}}\gamma^{-1}, \quad Q = -\pi Q\pi^{-1}. \quad (15)$$

The solution of these equations is

$$Q = \begin{pmatrix} & -M^\dagger \\ M & \end{pmatrix} \quad (16)$$

with a condition $M = \gamma_0 M^* \gamma_0^{-1}$, where the explicit matrix in the above equation denotes the time-reversal space of W.

The Lagrangian (14) has G=Sp(n)×Sp(n) symmetry. To see this, let us consider the transformation $Q \to gQg^{-1}$ which keeps the symmetry relations (15). It turns out that g should satisfy $\gamma = g\gamma g^{\mathrm{t}}$ and $\tau g\tau^{-1} = g$ as well as $gg^\dagger = 1$, and therefore g is explicitly given by

$$g = \begin{pmatrix} g_+ & \\ & g_- \end{pmatrix}, \quad (17)$$

where $g_\pm \in \mathrm{Sp}(n)$ is a $2n \times 2n$ matrix in the replica and the spin space of W. So far we have derived the Lagrangian as well as its symmetry group. To integrate out the fermi fields, it may be convenient to use the notations where the fermi fields Ψ and $\widetilde{\Psi}$ are column and row vector, respectively, as usual. Using Eq. (15), fermion part of the Lagrangian (14) is rewritten as $\mathcal{L}_{\mathrm{F}} = \widetilde{\Psi}(\mathcal{H}_0\otimes 1 - 1\otimes Q^{\mathrm{t}})\Psi = \widetilde{\Psi}1\otimes\gamma(\mathcal{H}_0\otimes 1 + 1\otimes Q)1\otimes\gamma^\dagger\Psi$. Transform the fields as $\widetilde{\Psi} \to \widetilde{\Psi}1\otimes\gamma^\dagger$ and $\Psi \to 1\otimes\gamma\Psi$. Then the Lagrangian is given by, in terms of the fields M and $\psi_\pm$,

$$\mathcal{L} = \frac{1}{g}\mathrm{tr}_{W_{\mathrm{rs}}}\left[M^\dagger M - \epsilon\gamma_0(M - M^\dagger)\right] + \mathcal{L}_{\mathrm{F}1} + \mathcal{L}_{\mathrm{F}2}, \quad (18)$$

where W_{rs} is the replica and the spin space of W and $\mathcal{L}_{\mathrm{F}j}$ describes the Lagrangian of the jth-node fermion defined by

$$\mathcal{L}_{\mathrm{F}1} = \bar{\psi}^1_+ i\bar{\partial}\psi^1_+ + \bar{\psi}^1_- i\partial\psi^1_- - \bar{\psi}^1_+ M^\dagger\psi^1_- + \bar{\psi}^1_- M\psi^1_+, \quad (19)$$

and $\mathcal{L}_{\mathrm{F}2} = \mathcal{L}_{\mathrm{F}1}(1 \to 2, x \leftrightarrow y)$. Here and hereafter, the identity matrices such as those in $\bar{\partial}\otimes 1$ and $1\otimes M$ are suppressed. The transformation laws of M and $\psi_\pm$ fields are

$$M \to g_- M g_+^\dagger, \quad \bar{\psi}_\pm \to \bar{\psi}_\pm g_\pm^\dagger, \quad \psi_\pm \to g_\pm\psi_\pm. \quad (20)$$

We have used a bit complicated basis for W in the definition of Ψ and $\widetilde{\Psi}$ in Eq. (10), since Q and g become simpler in this basis. On the other hand, it may cause a difficulty in computing the saddle points. We have known, however, from the ϵ-term in Eqs. (13) and (14) that ω serves as a "metric" in the extended auxiliary space. Usually we may choose a basis of the space W with a diagonal metric, owing to which we can assume that Q is also diagonal on the saddle points. In the present case, therefore, it is natural to assume that Q should have the same structure as ω, and hence $M_0 = v\gamma_0$ with real diagonal matrix $v = \mathrm{diag}(v_1, \cdots, v_n)$. Then the variation with respect to v after the integration over fermi fields tells that the saddle points are given by $v_\alpha = v_0 \sim \Lambda\exp(-\pi v_F v_\Delta/g)$, where Λ is a ultraviolet cut-off. This solution gives rise to an exponentially small density of state at the band-center. Now it is easy to identify the saddle point manifold as H=Sp(n): The chiral transformation $g \equiv (g_+, g_-)$ in Eqs. (17) and (20) is divided into two types. One is $g_{\mathrm{v}} = (g_1, g_1^*)$ under which M_0 is still invariant, and the other is $g_{\mathrm{a}} = (g_2, g_2^{\mathrm{t}})$ under which M_0 is no longer invariant.

Nonlinear sigma model is derived as small fluctuation around the saddle point manifold H by considering g_{a} type local Sp(n) rotation. Let us parameterize $M = \xi^* H\xi = \widetilde{H}U$ with $\xi \in \mathrm{Sp}(n)$, $\widetilde{H} = \xi^* H\xi^\dagger$, and $U = \xi^2$. The field ξ describes the massless fluctuation around the saddle point manifold H, whereas H describes massive longitudinal modes. In what follows, we take only the leading order for the latter mode, setting $H = M_0(= \widetilde{H})$. It should be noted that nonlinear sigma model on G/H has a global G symmetry as well as a hidden local H symmetry. Though the field ξ itself is not invariant under local H transformation, the composite field U is invariant.

To derive an effective action for the transverse mode, let us come back to the Lagrangian (19), since we should be careful in the integration over fermi fields. It is practical to firstly integrate out the fermi field of node-1. Then, the contribution from node-2 can be obtained by replacing $x \leftrightarrow y$. To carry out the former integration, make the transformation $\bar{\chi}_+ = \bar{\psi}_+ U^\dagger$ and $\chi_+ = U\psi_+$, whereas $\bar{\chi}_- = \bar{\psi}_- 1_n\otimes\sigma_2$ and $\chi_- = 1_n\otimes\sigma_2\psi_-$. The Lagrangian (19) is then converted into

$$\mathcal{L}_{\mathrm{F}1} = \bar{\chi}^1_+ i\bar{D}\chi^1_+ + \bar{\chi}^1_- i\partial\chi^1_- + iv_0\left(\bar{\chi}^1_+\chi^1_- + \bar{\chi}^1_-\chi^1_+\right), \quad (21)$$

where D_μ is defined by $D_\mu = \partial_\mu + L_\mu$ with $L_\mu = U\partial_\mu U^\dagger$. It is convenient to scale $x = v_F x'$ and $y = v_\Delta y'$ working in the node-1 sector. Integration over the fermi field of node-1 yields

$$Z_{\mathrm{F}1} = e^{-\frac{1}{2}\Gamma_1(U)}$$
$$\times\mathrm{Det}^{\frac{1}{2}}_{V_1\otimes W_{\mathrm{rs}}} 1_2\otimes\begin{pmatrix} iv_0 & i(-i\partial_1 - \partial_2) \\ i(iD_1 - D_2) & iv_0 \end{pmatrix}, \quad (22)$$

where the identity matrix 1_2 belongs to the spin space of V, V_1 means the node-1 sector of V, the derivatives are with respect to the scaled coordinates x' and y', and $\Gamma_1(U)$ is the Jacobian for the chiral transformation, which can be calculated by using the Fujikawa method [13]. We simply present the final answer $Z_{\rm F1} \sim e^{-S_1}$, where the effective action S_1 associated with the node-1 fermion is composed of the principal chiral action of U with a coupling constant $\lambda = 4\pi$ and of the WZW term

$$\Gamma_{\rm WZW} = \frac{i}{12\pi}\int d^3x\epsilon_{\mu\nu\tau}{\rm tr}_{W_{\rm rs}}\partial_\mu UU^\dagger\partial_\nu UU^\dagger\partial_\tau UU^\dagger. \quad (23)$$

Therefore, integration over fermion of node-1 actually yields the WZW term.

Next let us compute the contribution from the node-2. The procedure is the rescaling $x' = x/v_F$ and $y' = y/v_\Delta$ and the exchange $x \leftrightarrow y$. It should be noted that the WZW term is invariant under the rescaling but is odd under the exchange, and hence it cancels out. The total effective action ends up with

$$S = \int d^2x{\rm tr}_{W_{\rm rs}}\left[\frac{1}{2\lambda}\partial_\mu U\partial_\mu U^\dagger - \epsilon\left(U+U^\dagger\right)\right] + k\Gamma_{\rm WZW} \quad (24)$$

with $\frac{1}{\lambda} = \frac{1}{4\pi}\frac{v_F^2+v_\Delta^2}{v_Fv_\Delta}$ and $k = 0$. This is just the action derived by Senthil *et al.* Although the WZW term disappears in the ordinary d-wave superconductors, we are tempted to expose it, since the WZW term exists potentially. This is indeed possible: If the lattice model includes the symmetry breaking term (3) and therefore the Dirac fermion for the node-2 has a mass (7), we can neglect the node-2 fermion in the lower energy than the mass gap. In this case, we have the same action (24) but with $\frac{1}{\lambda} = \frac{1}{4\pi}$ and $k = 1$ in the scaled coordinates x' and y'.

The renormalization group equations of the action (24) are calculated at the one-loop order as

$$\frac{d\lambda}{d\ln L} = -\varepsilon\lambda + \frac{\lambda^2}{4\pi}\left[1-\left(\frac{k\lambda}{4\pi}\right)^2\right],$$
$$\frac{d\epsilon}{d\ln L} = \left(d - \frac{\lambda}{8\pi}\right)\epsilon, \quad (25)$$

where $d = 2$, $\varepsilon = d-2$, and the replica limit $n \to 0$ has been taken. In the case where $m = 0$, we reach the same conclusion as Senthil *et al.* Namely, one-quasiparticle states are localized, since the spin conductance against weak magnetic fields is related with λ as $\sigma = 2/(\pi\lambda)$, which is calculated [6] via diffusion constant in the diffusion propagator (9). It should be stressed that the cancellation of the WZW term is due to the four-fold symmetry of the d-wave Hamiltonian. Actually, the latent WZW term can emerge via symmetry breaking mass term for the Dirac fermions, and in that case the coupling constant λ flows to the strong-coupling fixed point value $\lambda_{\rm c} = 4\pi/k$. This fixed-point is conformal invariant described by ${\rm Sp}(n)$ WZW model. From the scaling dimension of the energy in Eq. (25), it turns out that the density of state near this fixed-point obeys the scaling law

$$\rho(E) = E^{\frac{1}{4k-1}}. \quad (26)$$

Therefore, if the pure model has the breaking term (3), we suggest that $\rho(E) = E^{\frac{1}{3}}$.

In this paper, we have taken the breaking term of type (3) into account. More detailed phase diagram will be published elsewhere. Numerical check of the present conjecture does not seem difficult. It is quite interesting to expose the hidden WZW term which exists potentially but conceals itself in the four-fold symmetry of the d-wave superconductors.

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